Gödel's Path from Hilbert and Carnap to Husserl

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It is known that the great logician Kurt Gödel began to study the work of Edmund Husserl in 1959 and that he continued to study Husserl's work until the end of his life. In this paper I shall discuss some developments in Gödel's work that led him to Husserl's philosophy.[1] The latter part of the paper in particular is focused on a short text, probably written in 1961, in which Gödel tells us explicitly how he sees Husserl's phenomenology in relation to the modern development of the philosophy and foundations of mathematics.

It is clear from remarks that he made to the logician Hao Wang and from notes in his *Nachlass* that Gödel was most interested in Husserl's transcendental eidetic phenomenology. In other words, he was interested in the conception of phenomenology that Husserl inaugurated during his Göttingen years and that was expressed in publication in mature form in *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie* (1913). In the first edition of his *Logische Untersuchungen* (*LU*) of 1900–1901 Husserl had characterized phenomenology as a type of descriptive psychology in the style of his teacher Franz Brentano but during the Göttingen years he developed his new transcendental phenomenology in which we are to 'bracket' the 'natural attitude' on the basis of the so-called phenomenological *epochē* (or 'reduction'). The *LU* of course provide extensive discussions of philosophical ideas about logic but Gödel says that what is missing in the *LU* is still a correct method. Hao Wang has written that

> According to Gödel, Husserl just provides a program to be carried out; his *Logical Investigations* is a better example of the execution of this program than is his later work, but it has no correct technique because it still adopts the "natural" attitude. (Hao Wang, *A Logical Journey: From Gödel to Philosophy*, p. 164)

When Gödel says in this quotation that the *LU* has no correct technique because it still adopts the 'natural' attitude it is to Husserl's shift to transcendental phenomenology that he is referring. It was this latter viewpoint that Gödel thought would be promising in logic, the foundations of mathematics, and in

1 There is much more to say than cannot be included in this paper. My lecture at the symposium *Edmund Husserl 1859-2009*, along with most of this paper, was drawn from chapter 3 of my forthcoming book *After Gödel: Platonism and Rationalism in Mathematics and Logic* (Oxford University Press). See also Part II of Tieszen 2005.

philosophy itself. In a note in the *Nachlass*, for example, Gödel refers to one of Husserl's earliest presentations of transcendental phenomenology in "Die Idee der Phänomenologie" (1907) as a 'momentous lecture'.

As we will see below, it is quite interesting how in 1961 Gödel juxtaposes his interest in Husserl with the work of another great Göttingen figure who was a senior colleague of Husserl during Husserl's Göttingen period, David Hilbert.

I. Gödel 1953 to 1961: Two Central Philosophical Texts

In May of 1953 the editor of the *Library of Living Philosophers* series, Paul A. Schlipp, wrote to Kurt Gödel to invite him to contribute a paper to a volume on the philosophy of Rudolf Carnap. After drafting six versions of the paper and engaging in several rounds of correspondence, Gödel wrote to Schilpp in February of 1959 to say that he was not satisfied with the paper and had therefore decided not to publish it. Hao Wang, writing about Gödel's decision to abandon the Carnap paper and to study Husserl's work, says that

> It seems to me that the two decisions may have been related. He had, he once told me, proved conclusively in this [Carnap] essay that mathematics is *not* syntax of language but said little about what mathematic *is*. At the time he probably felt that Husserl's work promised to yield convincing reasons for his own beliefs about what mathematics is. (Wang 1996, p. 163)

Continuing in this vein, Wang says

> It is, therefore, not surprising that, when he commented on various philosophers during his discussions with me, he had more to say about the views of Husserl than about the positivists or empiricists. Indeed, his own criticisms of the empiricists tend to be similar to Husserl's.

I want to follow up on these comments of Wang and focus on some elements of Gödel's critique of Carnap in the versions of his Carnap paper "Is Mathematics Syntax of Language?" (Gödel *1953/9) in order to show how these are linked to Gödel's discussion of Husserl's philosophy in "The Modern Development of Mathematics in the Light of Philosophy" (Gödel *1961/?). Investigation of the philosophical and conceptual links between these texts can help us to deepen our understanding of how Gödel saw Husserlian phenomenology and to indicate in greater detail what he hoped to obtain from his study of it.

II. Gödel's Arguments Against Carnap's Claim that Mathematics is Syntax of Language

Gödel says that Carnap's program in *Logische Syntax der Sprache* (*LSS*) and related works in the nineteen thirties aims to establish three basic philosophical points: (i) mathematical intuition (which is later associated by Gödel with Husserl's categorial intuition or *Wesensschau*), for all scientifically relevant purposes, can be replaced by conventions about the use of symbols and their application; (ii) mathematics, unlike other sciences, does not describe any existing mathematical objects or facts. Rather, mathematical propositions, because they are nothing but consequences of conventions about the use of symbols and are therefore compatible with all possible experience, are void of content; and (iii) the conception of mathematics as a system of conventions makes the *a priori* validity of mathematics compatible with strict empiricism (Gödel *1953/9-V, p. 356). There is, according to Carnap, a strict division between analytic and synthetic truths in which each kind of truth has its own place. Gödel says that his incompleteness theorems and some of his other mathematical results "tend to bring the falsehood of these assertions to light" (Gödel *1953/9-V, p. 356).

It is very interesting to note, in light of his later interest in Husserl's idea of philosophy as rigorous science, that Gödel should apply his incompleteness theorems to refute Carnap's view of mathematics. It can be argued that Gödel's incompleteness theorems can themselves be seen as examples of philosophy become rigorous science. They establish in a scientifically rigorous way, for example, that we cannot identify proof in formal systems with mathematical truth, and they thereby lead to a clarification of the meaning of the concepts of "formal proof" and "mathematical truth". As we will see below, Gödel is impressed with Husserl's claim that phenomenology offers a method for clarification of the meaning of concepts.

According to the logical positivism of Carnap there is a strict distinction between truths of mathematics/logic and empirical truths. It is allegedly possible to reconcile the *a priori* nature of mathematics and logic with empirical science by holding that the truths of mathematics and logic are based solely on linguistic (syntactical) conventions, while the truths of empirical science depend on verification in the world of sensory experience. The verificationist theory of meaning applies to sentences of empirical theories but not to sentences of mathematics and logic. Carnap says that the meaning of a sentence about reality (= empirical but not platonic reality) lies in its method of verification. If there is no possible method of verification in sense experience then sentences alleged to be about something are in fact meaningless. In works such as "Überwindung der Metaphysik durch logische Analyse der Sprache" (Carnap 1932) Carnap says that meaningful statements are in fact of the following kinds: first, there are statements that are true solely by virtue of their form. These are, following Wittgenstein, the tautologies. These say nothing about reality. The formulas of logic and mathematics are of this kind. They are not factual statements but serve for the transformation of factual statements. There are also negations of such statements, contradictions, which are false by virtue of their form. The decision about the truth or falsity of all other statements lies in protocol sentences. Protocol sentences are true or false empirical statements and belong to the domain of empirical science. Any statement one desires to construct that does not fall within these categories becomes automatically meaningless (Carnap 1932). Carnap says in various places that sentences of empirical theories thus have content, but sentences of mathematics and logic are without content or object (see Carnap 1935; also Gödel *1953/59-III, p. 335). As Gödel puts it, the logical positivist view of mathematics, developed under the influence of Wittgenstein, consists of a combination of conventionalism and nominalism. Sentences of mathematics are true solely on the basis of syntactical (linguistic) conventions (conventionalism), and the formal systems that embody these conventions are given in sense experience (i.e., empirically) as systems of symbols (nominalism).

In *LSS* Carnap indicates that his view of logical syntax is influenced by Hilbert. Carnap says that a theory, rule, definition or the like is to be called 'formal' when no reference is made in it to the meanings of symbols. The only reference is to the kinds and order of the symbols from which the expressions are constructed (*LSS*, § 1). Carnap extends this idea into a philosophical position in its own right, based on ideas that are not found in Hilbert's writings but that were afoot in the Vienna Circle. The view in *LSS* is that it depends entirely on the formal structure of particular language and of the sentences involved whether a sentence should count as 'analytic' or not. Analytic sentences, including the sentences of mathematics and logic, are not about anything (*LSS*, § 2). Gödel's own view of analyticity is quite different from this, as we will see below. From the point of view of phenomenology, what we have here is a claim about intentionality in mathematics and logic. Carnap is claiming that empirical sentences are *about* something but sentences of logic and mathematics are not. Analytic sentences are without content or object. All of this applies, in particular, to sentences of mathematics that seem to be about numbers of various kinds, sets, functions, groups, spaces, and so on.[2]

Gödel's idea of applying his second incompleteness theorem to refute Carnap's view is rather clever: in order for the truths of mathematics to be based solely on linguistic (syntactical) conventions the syntactical conventions must be consistent. For if they are not consistent then all statements will follow from them, including all factual (empirical) statements. A rule about the truth of sentences can be called syntactical only if it does not imply the truth or false-

2 On the face of it, Carnap's view that mathematical propositions are not about anything seems quite implausible. This is perhaps why Gödel told Wang (Wang 1996, p. 174) that: "Carnap's work on the nature of mathematics was remote from actual mathematics; he later came closer to actual science in his book on probability".

hood of any "factual" sentence, i.e., one whose truth depends on extralinguistic facts. This requirement follows from the concept of a convention about the use of symbols but also from the fact that it is the lack of content of mathematics upon which its a priori nature, in spite of strict empiricism, is supposed to depend.

Therefore, a consistency proof for the syntactical conventions is required. Now we apply the second incompleteness theorem. This theorem should be applicable because the logical positivist will want classical mathematics or at least the mathematics required for physics and natural science. According to the second theorem, no formal system in which it is possible to do elementary arithmetic will, if it is consistent, contain the resources required to prove its own consistency. A consistency proof for any such sets of syntactical conventions will require objects, concepts, or methods that are not part of the systems under consideration. Gödel considers two possibilities for such a consistency proof: either it will be mathematical in nature or empirical and inductive in nature.

Suppose the consistency proof is mathematical in nature. In order for mathematics to be syntax of language in Carnap's sense it will have to be required that "language" will mean some symbolism that can actually be exhibited and used in the empirical world. In particular, its sentences will have to consist of a finite number of symbols, since sentences of infinite length do not exist in and cannot be produced in the empirical world. (The latter kinds of sentences, were they to exist, would presumably have to be purely mathematical objects.) Similarly, the "rules of syntax" will have to be finitary and cannot contain phrases such as "If there exists an infinite set of expressions with a certain property" for the simple reason that such phrases could not be finitarily meaningful. Not only must the rules of syntax be finitary, but in the derivation of axioms from them and in the proof of their consistency only finitary syntactical concepts can be used, i.e., only concepts referring to finite combinations of symbols. Now the second incompleteness theorem, as we just noted, tells us that no formal system in which it is possible to do elementary arithmetic will, if it is consistent, contain the resources required to prove its own consistency. According to this theorem, therefore, what will be required for the consistency proof will not be finitary, completely given empirically, and so on. Rather, it will be non-finitary, and will not be completely given empirically. If we want a *mathematical* consistency proof then the proof will involve 'abstract' and infinitary objects, concepts, or methods into which we have some insight. If we are to have a consistency proof at all then we will need some mathematical principles into whose content we have insight (intuition), whose meaning we understand. Otherwise there is no hope for a consistency proof. In this case, mathematical intuition, distinctive mathematical content, the abstract, and the non-finitary, are all back in the philosophical picture.

Gödel says that in order for mathematical intuition and the assumption of mathematical objects or facts to be dispensed with by means of syntax it will have to be required that the use of "abstract" and "transfinite" concepts of mathematics that can be understood or used only as a consequence of mathematical intuition or of assumptions about their properties can be replaced by considerations about finite combinations of symbols. However,

> If, instead, in the formulation of the syntactical rules some of the very same abstract or transfinite concepts are being used - or in the consistency proof, some of the axioms usually assumed about them - then the whole program completely changes its meaning and is turned into its downright opposite: instead of clarifying the meanings of the non-finitary mathematical terms by explaining them in terms of syntactical rules, non-finitary terms are (used) in order to formulate the syntactical rules; and instead of justifying the mathematical axioms by reducing them to syntactical rules, these axioms (or at least some of them) are necessary in order to justify the syntactical rules (as consistent). (Gödel *1953/59-III, pp. 341-342)

The conventionalism and nominalism of the logical positivist thus "changes its meaning and turns into its downright opposite". In this case, as Gödel indicates, we are also led to a conception of meaning clarification that is very different from Carnap's conception. Gödel discusses an alternative view of meaning clarification due to Husserl in more detail in the 1961 text, as we will see below.

On the other hand, suppose the consistency 'proof' is empirical in nature. In this case the claim to consistency is based on the fact that the conventions have thus far (in our use of them) not been found to lead to inconsistency. The evidence for consistency is based on past experience, i.e., it is inductive evidence. In this case we have to rely on empirical facts to sustain syntactical conventionalism about mathematical truths, i.e., to support the claim that the syntactical conventions are consistent in order to prevent all statements, including factual statements, from following from the conventions. This reliance on empirical evidence or empirical facts to maintain syntactical conventionalism about mathematical truths again violates the claim that the latter truths should be based solely on syntactical (linguistic) conventions, come what may in the empirical world. Furthermore, the empirical assertions used to support the consistency claim in this case would have content, so that content will again be required, albeit empirical (as opposed to mathematical) content. In short, one would have to appeal to sentences with content in order to be able to hold to strict conventionalism about mathematical statements that are supposed to have no content. Under this alternative mathematical statements completely lose their a priori character, their character as linguisitic conventions, and their voidness of content. Thus, we can again not hold to strict linguistic conventionalism about mathematics.

In sum, it is not possible without a consistency proof to be a conventionalist/nominalist about mathematics in the manner of Carnap's early logical positivism, but what is needed for the consistency proof, whether it is mathematical or empirical in nature, undermines the conventionalism and nominalism of the logical positivists.[3]

What could *not* be true about mathematics if Gödel's critique of Carnap is correct? We could not replace mathematical (later, categorial or "eidetic") intuition with conventions about the use of symbols and their application if we want a mathematical consistency proof for syntactical systems. Similarly, it could not be the case that mathematics does not describe any existing objects or facts. Mathematical propositions could not be empty tautologies that are void of content. The truths of mathematics could not be based solely on linguistic conventions. Rather, true mathematical propositions and true empirical propositions would both be about objects or facts, and there would be analogies in our knowledge of such objects, even if the objects were of different types. Gödel says, as Quine does later, that the logical positivist's way of drawing the distinction between the a priori and the a posteriori, the mathematical and the empirical, the analytic and the synthetic, will not work. Gödel does not, however, adopt a Quinean holism about this matter. Instead, he says that there is one ingredient of Carnap's incorrect theory of mathematical truth that is correct and discloses the true nature of mathematics, namely, it is correct that a (pure) mathematical proposition says nothing about physical or psychical reality existing in space and time, because it is already true owing to the meaning of the terms occurring in it, irrespective of the world of real things. What is wrong with Carnap's view, Gödel says, is that the meaning of the terms (that is, the concepts they denote) is asserted to be something man-made and consisting merely of linguistic conventions.

> The truth, I believe, is that these concepts form an objective reality of their own, which we cannot create or change, but only perceive and describe. Therefore, a mathematical proposition, although it does not say anything about space-time reality, still may have a very sound objective content, insofar as it says something about relations of concepts. (Gödel ⋆1951, p. 320)

Gödel thus contrasts his own view of analyticity with Carnap's: "analytic" does not mean "true owing to our definitions", but rather it means "true owing to the nature of the concepts" occurring in mathematical statements (Gödel ⋆1951, p. 321).

One way of holding, against platonism, that mathematics is our own free creation or free invention is to construe "free creation" in terms of linguistic conventions in the style of logical positivism (Gödel ⋆1951). Gödel thinks that he has offered powerful arguments against this kind of anti-platonism. There might be some room for free creation in mathematics, but Gödel says in a number of places that there is no such room in the case of the primitive concepts. In the Gibbs Lecture (Gödel ⋆1951) this is of course taken up in more

3 There have been several attempts to defend Carnap against Gödel's argument. In my view, there are a number of problems with these attempts but I do not have space to consider them here.

detail. Psychologism and Aristotelian realism about mathematics are also rejected in the 1951 Gibbs Lecture.

Before proceeding to the next section I want to note that in his drafts of the Carnap paper Gödel is responding to a number of Carnap's views on meaning clarification, the methodology of logical analysis, and metaphysics. The language of meaning clarification, in particular, is prominent in the 1961 text of Gödel that we will consider in a moment. Carnap says that modern logic has made it possible to give a new and sharper answer to the question of the validity and justification of metaphysics. Research in applied logic or theory of knowledge aims at clarifying the cognitive content of scientific statements and, thereby, the meanings of the terms that occur in such statements. It is on this basis that metaphysics can be eliminated (Carnap 1932). What is left for philosophy if metaphysics is to be eliminated? What remains, Carnap says, is a method, the method of 'logical analysis', i.e., laying out the syntax of the language of science. This method, in its negative use, will serve to eliminate meaningless expressions. In its positive use, it serves to clarify meaningful concepts and propositions, and to lay logical foundations for factual science and for mathematics. It is only logical analysis in this sense that can count as "scientific philosophy". We already noted above how, on the basis of the application of the second incompleteness theorem, Gödel suggests a reversal of Carnap's view: instead of clarifying the meanings of the non-finitary mathematical terms by explaining them in terms of syntactical rules, non-finitary terms are used in order to formulate the syntactical rules; and instead of justifying the mathematical axioms by reducing them to syntactical rules, these axioms (or at least some of them) are necessary in order to show that the syntactical rules are consistent.[4] In the text to which we will now turn we will see very clearly, if only in outline, the alternative methodology of meaning clarification and the alternative conception of "scientific philosophy" that Gödel thinks we find in Husserl's work.

III. The Modern Development of the Foundations of Mathematics in the Light of Philosophy

In Gödel's 1961 text Carnap is not mentioned by name. Gödel does mention positivism as a philosophy that falls on the 'left' side of his schema of possible

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4 Gödel made the following comment to Hao Wang: "Some reductionism is right: reduce to concepts and truths, but not to sense perceptions. Really it should be the other way around: Platonic ideas ... are what things are to be reduced to. Phenomenology makes them [the ideas] clear" (Wang 1996, p. 167). The "reduction" Gödel mentions here would presumably be subsumed under the phenomenological epochē. In the case of mathematics and logic empiricist reductionism is contrasted with the phenomenological reduction.

philosophical worldviews but he focuses mostly on the 'leftward' foundational program of Hilbert. Gödel notes some relationships between the views of Hilbert and Carnap in several places. In Gödel *1953/9 III, footnote 19, for example, he says that he thinks that if the syntactical program of Carnap is to serve its purpose then what must be understood by "syntax" is equivalent to Hilbert's finitism, in the sense that it consists of those concepts and forms of reasoning, referring to finite combination of symbols, which are contained within the limits of "that which is directly given in sensual intuition". In Gödel *1951 (p. 315) he says that the formalistic foundation of mathematics would be a special elaboration of Carnap's syntactical view and that, on the other hand, it turns out that the feasibility of the nominalistic program of the logical positivists implies the feasibility of the formalistic program. In the 1961 text to which I will now turn, Husserl's philosophy is set up as an alternative to both positivism and Hilbert's foundational view. Gödel links Husserl to Kant at the end of the 1961 text, as we will see, but he also mentions how some modifications of Kant's view will be required.

In his 1961 text Gödel sets up a general schema of possible philosophical worldviews according to their degree and manner of affinity to metaphysics. We obtain a division into two groups, with skepticism, materialism and positivism on one side and spiritualism, idealism, and theology on the other. If one thinks of philosophical doctrines as arranged along a line from left to right in this manner then empiricism belongs on the left side and a priorism belongs on the right. Pessimism belongs on the left side and optimism in principle toward the right, for empiricist skepticism is a kind of pessimism with regard to knowledge. Materialism is inclined to regard the world as an unordered and therefore meaningless heap of atoms. On the other hand, idealism and theology see meaning, purpose, and reason in everything. Additional examples of theories on the right side would include theories of objective moral values and objective aesthetic values, whereas the interpretation of ethics and aesthetics on the basis of custom, upbringing and so on would fall on the left.

Gödel says that the development of philosophy since the Renaissance has, on the whole, gone from right to left. This development has also made itself felt in mathematics. Mathematics, as an a priori science, always has an inclination toward the right and has long withstood the *Zeitgeist* that has ruled since the Renaissance. The empiricist conception of mathematics, such as that set forth by John Stuart Mill in the nineteenth century, did not find much support. Indeed, mathematics evolved into ever higher abstractions, away from matter and into ever greater clarity in its foundations. The foundations of the infinitesimal calculus and the complex numbers, for example, were improved. Mathematics thus moved away from skepticism. Around the turn of the century, however, the antinomies appeared in mathematics. Gödel says that the significance of the antinomies was exaggerated by skeptics and empiricists and that the antinomies were employed as a pretext for a leftward upheaval. He says, in response, that the contradictions did not appear in the heart of mathematics but rather near its outer boundary toward philosophy. Moreover, the antinomies have been resolved in a manner that is satisfactory to those who understand set theory.[5] These kinds of points are of no use, however, against the prevailing *Zeitgeist*. Many mathematicians came to deny that mathematics as it had developed previously represented a system of truths. They acknowledged this for a larger or smaller part of mathematics only and retained the rest in at best a hypothetical sense.

Now Carnap's view of mathematics as syntax of language is clearly on the left in Gödel's schema, and the arguments in the drafts of the Carnap paper are meant to refute it. Gödel says that the 'nihilistic' consequences of the spirit of the times also led to a reaction in mathematics itself. Thus came into being "that curious hermaphroditic thing that Hilbert's formalism represents". It sought to do justice both to the *Zeitgeist* and to the nature of mathematics. In conformity with the ideas prevailing in recent philosophy, it acknowledges that the truth of the axioms from which mathematics starts cannot be justified or recognized in any way and therefore that the drawing of consequences from them has meaning only in a hypothetical sense. The drawing of consequences itself, to further satisfy the spirit of the time, is construed as a mere game with symbols according to rules, where this is likewise not thought of as supported by insight or intuition.[6] In accord with the earlier 'rightward' philosophy of mathematics and the mathematicians instinct, however, it is held that a proof of a proposition must provide a secure grounding for the propositions and that

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5 In some places in his writings Gödel likens the antinomies to illusions of the senses. They are cases where we have not seen concepts clearly enough, and they lose their grip on us once we have achieved more clarity in our perception of concepts. See, for example, the passages in Wang 1974 on perceiving concepts clearly (with references to Husserl), pp. 81-86. Also, Gödel *1953/9, p. 321, and 1964, p. 268.

6 In various places in his writings, going back to the thirties, Gödel distinguishes the purely formalistic and relative concept of proof from the concept of proof as "that which provides evidence". See, e.g., Gödel 193?, p. 164, where Gödel says that the incompleteness theorems show that in the transition from evidence to formalism something is lost, and that the incompleteness theorems therefore do not undermine Hilbert's conviction in the solvability of every precisely formulated mathematical question. The concept of proof as that which provides evidence is said to be an "abstract concept" of proof. Elsewhere, he gives as an example of an "abstract concept" the concept of proof, understood in the non-formalistic sense of "known to be true". A concept is said to be "abstract" if it does not refer to sensory objects (see, e.g., Gödel *1951, p. 318, footnote 27). A similar view is expressed in footnote 20 of Version III of the Carnap paper (Gödel *1953/59, p 341), where Gödel says that the concept of proof , in its original contentual meaning, is an abstract concept. This is the concept according to which a proof is not "a sequence of expressions satisfying certain formal conditions, but a sequence of thoughts convincing a sound mind". Here Gödel says that the abstract and the transfinite concepts together form the class of "non-finitary" concepts. See also Gödel 1972, p. 273, footnote e, where Gödel refers to the concept "p implies q" as an "abstract concept" when it is understood in the sense of "From a convincing proof of p a convincing proof of q can be obtained".

every precisely formulated yes-or-no question in mathematics must have a clear cut answer. One aims to prove, that is, for the inherently unfounded rules of the game with symbols that of two sentences A and ¬ A, exactly one can always be derived. Such a system is consistent if not both can be derived, and if one can be derived then the mathematical question expressed by A can be unambiguously answered. In order to justify the assertions of consistency and completeness a certain part of mathematics must be acknowledged to be true in the sense of the old rightward philosophy. The part in question, however, is much less opposed to the spirit of the time than the high abstractions of set theory. It is the part that refers only to concrete and finite objects in space, namely the combinations of symbols. This is Hilbert's finitistic formalism. In Hilbert's program we thus see an interesting mixture of rationalist and empiricist elements.

The next step in the development comes with Gödel's incompleteness theorems: it turns out that it is impossible to rescue the older rightward aspects of mathematics in such a way as to be in accord with the spirit of the time. Even for elementary arithmetic it is impossible to find a system of axioms and formal rules from which, for every number-theoretic proposition A, either A or ¬ A would always be derivable. Moreover, for reasonably comprehensive axioms of mathematics it is impossible to provide a proof of consistency merely by reflecting on the concrete combinations of symbols, without introducing more abstract elements.[7] Hilbert's combination of materialism and aspects of classical mathematics thus proves to be impossible. This means that the combination of rationalist and empiricist elements involved in Hilbert's program is unworkable. It is not possible to be a finitisitic formalist and to hold that every clearly stated mathematical proposition is decidable. Nor is it possible to hold that proofs provide a secure grounding of mathematical propositions, in the sense that they provide *evidence* for those propositions.

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7 See also Gödel's remark in Gödel 1972, p. 271-273: "P. Bernays has pointed out on several occasions that, in view of the fact that the consistency of a formal system cannot be proved by any deduction procedures available in the system itself, it is necessary to go beyond the framework of finitary mathematics in Hilbert's sense in order to prove the consistency of classical mathematics or even of classical number theory. Since finitary mathematics is defined as the mathematics of *concrete intuition*, this seems to imply that *abstract concepts* are needed for the proof of consistency of number theory ... [What Hilbert means by "Anschauung" is substantially Kant's space-time intuition confined, however, to configurations of a finite number of discrete objects.] By abstract concepts, in this context, are meant concepts which are essentially of the second or higher level, i.e., which do not have as their content properties or relations of *concrete objects* (such as combinations of symbols), but rather of *thought structures* or *thought contents* (e.g., proofs, meaningful propositions, and so on), where in the proofs of propositions about these mental objects insights are needed which are not derived from a reflection upon the combinatorial (space-time) properties of the symbols representing them, but rather from a reflection upon the *meanings* involved."

Gödel says that only two possibilities remain. Either we must give up the older rightward aspects of mathematics or attempt to uphold them in contradiction to the spirit of the time. The first alternative suits the *Zeitgeist* and is therefore usually the one adopted. One has to thereby give up on features of mathematics that would otherwise be very desirable, namely, to (i) safeguard for mathematics the certainty of its knowledge by thinking of proof as that which provides evidence but also to (ii) uphold the optimistic belief that for clear questions posed by reason it is possible for reason to find clear answers.[8] One would give up on these features not because any mathematical results compel us to do so but because this is the only way to remain in agreement with the prevailing philosophy. Gödel grants that great advances have been made on the basis of the leftward spirit in philosophy and he thinks there have been excesses and wrong directions taken in the preceding rightward philosophies. The correct attitude is that the truth lies in the middle of these philosophies or consists in a combination of the leftward and rightward views, but not in the manner of Hilbert's conception. Hilbert's combination, like Carnap's, was too primitive and tended too strongly in one direction. We must look elsewhere for a workable combination. If we want to preserve elements of the earlier rightward view of mathematics then we must suppose that the certainty of mathematics is not to be secured by proving certain properties by a projection onto material systems (i.e., the mechanical manipulation of physical symbols) but rather by cultivating and deepening our knowledge of the abstract concepts that lead to setting up these mechanical systems in the first place. Furthermore, it is to be secured by seeking, according to the same procedures, to gain insights into the solvability of all meaningful mathematical problems.

How is it possible to extend our knowledge of these abstract concepts? How can we make these concepts precise and gain a comprehensive and secure insight into the fundamental relations that hold among them; that is, into the axioms that hold for them? We cannot do this by trying to give explicit definitions for concepts and proofs for axioms, since in that case one needs other undefinable abstract concepts and axioms holding for them. Otherwise one would have nothing from which one could define or prove. Therefore, the procedure must consist to a large extent in a clarification of meaning that does not consist in giving definitions. We thus see here, as in other places in his writing, that Gödel is speaking about the need to reflect on meaning. What is required is a reflection on meaning or on concepts that is of a 'higher level'

8 Wang 1974, pp. 324-25, reports that Gödel thought Hilbert was correct in rejecting the view that there exist number theoretic questions undecidable for the human mind because if it were true it would mean that human reason is utterly irrational in asking questions it cannot answer while asserting emphatically that only reason can answer them. Human reason would then be very imperfect and, in some sense, even inconsistent, which contradicts the fact that those parts of mathematics that have been systematically and completely developed (such as the theory of 1st and 2nd degree Diophantine equations) show an amazing degree of beauty and perfection.

than reflection on the combinatorial properties of concrete symbols. This is the kind of ascent that is a function of reason. The notion of meaning clarification here is thus quite unlike Carnap's notion, and it involves, on the basis of the second incompleteness theorem, just the kind of reversal of the sensory and the abstract, or the sensory and the categorial, that we quoted from *1953/9 paper above.

In looking for a workable combination of the two directions Gödel turns to the philosophy of Husserl. He says that there exists today the beginning of a science that claims to possess a systematic method for such a clarification of meaning and that is the phenomenology founded by Husserl. The conception of scientific philosophy here, however, is quite different from Carnap's conception. It is a conception of philosophy as a rigorous universal eidetic discipline, which Husserl portrays in some of his writings as an updated phenomenological version of a Leibnizian rationalism. Clarification of meaning, Gödel says, "consists in focusing more sharply on the concepts concerned by directing our attention in a certain way, namely onto our own acts in the use of these concepts, onto our powers in carrying out our acts, and so on." Phenomenology is not supposed to be a science in the same sense as other sciences. Rather, it is supposed to be a procedure that should produce in us a new state of consciousness in which we describe in detail the basic concepts we use in our thought, or grasp other basic concepts hitherto unknown to us.[9] Gödel says that he sees no reason to reject such a procedure at the outset as hopeless. Empiricists in particular have no reason to do so since that would mean that their empiricism is a kind of dogmatic apriorism.

One can in fact present reasons in favor of the phenomenological approach. Gödel's example of this in the 1961 paper is that if one considers the development of a child one sees that it proceeds in two directions. On the one hand the child experiments with objects in the external world and with its own sensory and motor organs. On the other hand, it comes to a better and better understanding of language and of the concepts on which language rests. Concerning this second direction we can say that the child passes through states of consciousness of various heights. A higher state is attained, for example, when the child first learns the use of words and, similarly, when for the first time it understands a logical inference. We can view the whole development of empirical science as a systematic and conscious extension of what the child does when it develops in the first direction. The success of this procedure is astonishing and far greater than one might expect a priori. After all it leads to

9 There are reference elsewhere in Gödel's thinking to meaning clarification and phenomenology. In Wang 1974, p. 189, for example, Wang says "With regard to setting up the axioms of set theory (including the search for new axioms), we can distinguish two questions, viz., (1) what, roughly speaking, the principles are by which we introduce the axioms, (2) what their precise meaning is and why we accept such principles. The second question is incomparably more difficult. It is my impression that Gödel proposes to answer it by phenomenological investigations."

the remarkable technological development of recent times. Gödel reasons that this makes it seem quite possible that a systematic and conscious advance in the second, rationalistic direction might also far exceed the expectations one might have a priori.[10]

There are examples that show how considerable further development in the second direction occurs even without the application of a systematic and conscious procedure, a development that transcends 'common sense'. Gödel's example here is that in the systematic establishment of axioms of mathematics new axioms that do not follow by formal logic alone from those previously established again and again become *evident*. His own incompleteness theorems could be used to show this, in the sense that we can augment a given formal system with its Gödel sentence, and then repeat this process indefinitely. He also has in mind the addition of more and more axioms of infinity in set theory. Gödel says that the incompleteness theorems, which are often viewed as negative results, do not exclude the possibility that every clearly posed mathematical yes-or-no question is solvable in this way, for it is just this becoming evident of more and more new axioms on the basis of the meaning of the primitive concepts that a machine cannot emulate.[11]

In the 1961 text Gödel goes on to say that there is an intuitive grasp of ever newer axioms that are logically independent of the earlier ones and that this

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10 In his letter to Rappaport (Gödel 1962, pp. 176-77) Gödel says that the developments in this direction, however, have not gone well: "Our knowledge of the abstract mathematical entities themselves (as opposed to the *formalisms* corresponding to them) is in a deplorable state. This is not surprising in view of the fact that the prevailing bias even denies their existence."

11 One of the rigorously proved results about minds and machines, based on his incompleteness theorems, according to Gödel, is that either the human mind surpasses all machines (i.e., it can decide more number theoretic questions than any machine) or there exist number theoretic questions undecidable for the human mind. This disjunction is discussed at some length in Gödel *1951. It appears in other places in Gödel's writing, and is also reported Wang 1974, pp. 324-25. The comments here should be compared with remarks Gödel makes elsewhere on abstract concepts, meaning, intuition, evidence, and decidability by human reason vs. mechanical decidability, e.g., "The generalized undecidability results do not establish any bounds for the powers of human reason, but rather for the potentialities of pure formalism in mathematics... Turing's analysis of mechanically computable functions is independent of the question whether there exist finite *non-mechanical* procedures ... such as involve the use of abstract terms on the basis of their meaning" (Gödel 1934, p. 370).
A similar theme is sounded in the following passage, although Gödel adds that human reason is capable of constantly developing its understanding of the abstract terms: "Turing in his 1937 ... gives an argument which is supposed to show that mental procedures cannot go beyond mechanical procedures. However, this argument is inconclusive. What Turing disregards completely is the fact that *mind, in its use, is not static, but constantly developing*, i.e., that we understand abstract terms more and more precisely as we go on using them, and that more and more abstract terms enter the sphere of our understanding" (Gödel 1972a, p. 306).

is necessary for the solvability of mathematical problems. He says that this agrees in spirit if not in letter with the Kantian conception of mathematics in the following sense: Kant asserted that in the derivation of geometrical theorems we always need new geometrical intuitions and that a purely logic derivation from a finite number of axioms is therefore impossible. Given what we now know about elementary geometry this is demonstrably false. But if we replace the term 'geometrical' by 'mathematical' or 'set-theoretical' then it becomes a demonstrably true proposition. What Gödel does not say here is that in this case we presumably need a conception of intuition, such as categorial intuition, that goes beyond Kant's (two forms of) sensory intuition. For Kant, (Euclidean) geometry was the form of outer sensory intuition.

Gödel remarks that many of Kant's assertions are false if understood literally but in a broader sense contain deep truths. The whole phenomenological method, according to Gödel, goes back in its central idea to Kant. What Husserl did was to formulate it more precisely, made it fully conscious, and actually carried it out for particular domains. It is because in the last analysis Kantian philosophy rests on the idea of phenomenology, albeit not in an entirely clear way, that Kant has had such an enormous influence over the entire development of philosophy. Quite divergent directions have developed out of Kant's thought, however, due to the lack of clarity and literal incorrectness of many of his formulations. None of these have really done justice to the core of Kant's thought. It is Husserl's phenomenology that for the first time meets this requirement. It avoids both the death defying leaps of idealism into a new metaphysics as well as the positivistic rejection of all metaphysics. The 1961 text concludes with a rhetorical question: if the misunderstood Kant has already led to so much that is interesting in philosophy and also indirectly in science, how much more can we expect from Kant correctly understood by way of Husserl?

IV. Conclusion

In the texts we have considered we see how Gödel argues against Carnap's empiricism, nominalism, and conventionalism about mathematics and against the 'leftward' aspects of Hilbert's program, namely Hilbert's finitistic formalism. In Husserl's transcendental eidetic phenomenology he finds a philosophical position that prizes the idea of clarification of the meaning of basic concepts and that recognizes abstract meanings (concepts) and categorial intuition, and he holds that this is the kind of outlook that is needed if we are to find consistency proofs, to preserve the idea of proof as providing evidence, and to decide undecidable Gödel sentences and open mathematical problems. A purely mechanical, formal or syntactic conception of provability will not suffice. What (Turing) machines or computers cannot emulate is the fact that more and more axioms become evident to us on the basis of the meaning of the primitive concepts of mathematics. The two 'rightward' elements that Gödel is concerned to preserve, which are also included in Hilbert's program but are incompatible (given the incompleteness theorems) with his finitistic formalism, include (i) optimism about deciding open mathematical problems by human reason and (ii) the claim that proofs should provide a secure grounding for mathematical propositions. Both of these 'rightward' elements can be retained if we shift to what Gödel calls the 'abstract' concept of proof, i.e., the concept of proof as that which provides evidence, and turn to ideas in Husserl's philosophy. It is in Husserl's work that we might find a workable combination of leftward and rightward elements. On the rightward side, however, how can we avoid a "death-defying leap of idealism" into a dubious metaphysics? Gödel evidently hopes that we can avoid the one-sidedness or prejudices of the positive sciences and yet steer clear of questionable metaphysical views by developing a scientific philosophy that employs the phenomenological *epochē* and still requires that knowledge in mathematics and logic depends not only on mere conception but also on intuition, only now it is categorial intuition or *Wesensanschauung* that we must cultivate.

References

Carnap, R. (1932) "Überwindung der Metaphysik durch logische Analyse der Sprache", E*rkenntnis* 2, 220-241.

— (1934) *Logische Syntax der Sprache*. Vienna: Springer.

— (1935) "Formalwissenschaft und Realwissenschaft", *Erkenntnis* 5, 30-37.

Gödel, K. (193?) "Undecidable Diophantine Propositions", in *Kurt Gödel: Collected Works*, Vol. III, 164-175.

— (1934) "On Undecidable Propositions of Formal Mathematical Systems" in *Kurt Gödel: Collected Works*, Vol. I, 346-371.

— (*1951) "Some Basic Theorems on the Foundations of Mathematics and their [Philosophical] Implications", in *Kurt Gödel: Collected Works*, Vol. III, 304-323.

— (*1953/9) III and V. "Is Mathematics Syntax of Language?", in *Kurt Gödel: Collected Works*, Vol. III, 334-363.

— (*1961/?) "The Modern Development of the Foundations of Mathematics in the Light of Philosophy", in *Kurt Gödel: Collected Works*, Vol. III, 374-387.

— (1962) "Letter to Leon Rappaport", in *Kurt Gödel: Collected Works*, Vol. V, 176-177.

— (1972) "On an Extension of Finitary Mathematics Which Has Not Yet Been Used", in *Kurt Gödel: Collected Works*, Vol. II, 271-280.

— (1972a) "Some Remarks on the Undecidability Results", in *Kurt Gödel: Collected Works*, Vol. II, 1990, 305-306.

Husserl, E. (1900) *Logische Untersuchungen*, Erster Theil: Prolegomena zur reinen Logik. Halle: M. Niemeyer. 2nd edition 1913.

— (1901) *Logische Untersuchungen*, Zweiter Theil: Untersuchungen zur Phänomenologie und Theorie der Erkenntnis. Halle: M. Niemeyer. 2nd edition in two parts 1913/22.

— (1907) "Die Idee der Phänomenologie". Published in 1950 in Husserliana II.

— (1913) "Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie", *Jahrbuch für Philosophie und phänomenologische Forschung* 1, 1-323.

Tieszen, R. (2005) *Phenomenology, Logic, and the Philosophy of Mathematics*. Cambridge University Press.

Wang, H. (1974) *From Mathematics to Philosophy*. New York: Humanities Press.

— (1987) *Reflections on Kurt Gödel*. Cambridge, MA: MIT Press.

— (1996) *A Logical Journey: From Gödel to Philosophy*. Cambridge, Mass.: MIT Press.